

# **DOE-FIU SCIENCE & TECHNOLOGY WORKFORCE DEVELOPMENT PROGRAM**

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# **Temperature Profiles for Single Shell Tank Closure through Mass Grout Pours**

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## ABSTRACT

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This report documents the results of an investigation of the expected temperature rise in large grout pours planned for closure of the single-shell tanks at DOE's Hanford Site. Grout testing was performed in 2009 to optimize grout formulations and collect data to aid future tank closure planning. Two grout formulations were developed with adequate self consolidating properties. One being the stabilizing grout mixture and the other being the bulk fill mixture. These grout mixtures were designed to be used for 75ft diameter Single Shell Tank (SST) Closure. With such a massive pour one must consider the rise in temperature associated with the heat of hydration. The objective of this investigation was to determine if the maximum temperature expected during tank closure using the bulk fill grout formulation was below levels of concern and thereby reducing or eliminating operational constraints associated with the temperature rise in the grout pour. After a diligent search for software, tools & techniques capable of providing reliable predictions we decided on using a piece of freeware named ConcreteWorks V2. Two different geometries within ConcreteWorks model predictions were made and compared to experimental data obtain during grout lift height experiments. Rectangular footings provided the best results and were then used to model large size footings much larger than an SST tank. Results indicated that maximum temperatures did not exceed 126°F. Although the maximum temperature deviations did exceeded 35°F it is unlikely that cracking will be an issue. If cracking becomes a serious concern then more research can be conducted.

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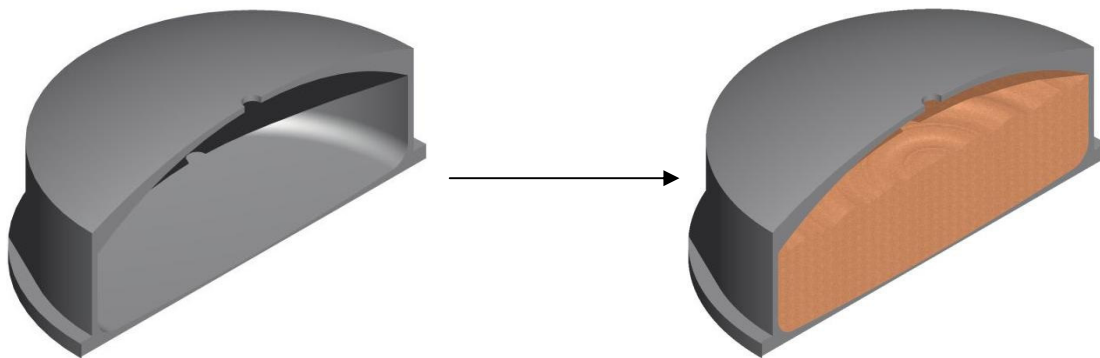
## 1. INTRODUCTION

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As society becomes more involved in environmental issues it becomes apparent that in order to improve the future we must deal with the past. Nowhere is this truer than at the Hanford Site located in south-central Washington. The Hanford Site was established during WWII as part of the Manhattan project and is officially the first plutonium production facility in world. The site served in our nation's defense for over forty years. Operations at the site have left approximately fifty-five million gallons of radioactive mixed waste stored in 177 underground tanks, 149 of them being single shell tanks (SST) and 28 being double shell tanks. These SSTs were built between 1943 and 1964. Due to mechanical weathering and extended use, some tanks have leaked and others are in danger of potential leakage (Roelant, 2007). There are plans for tank closure underway involving the removal and treatment of waste followed by stabilization of the tank structures.

One viable option for tank closure would be filling the tanks with cementitious material, such as grout (Figure 1). To accomplish this, a grout formulation optimization study was conducted by Columbia Energy and Environmental Services (CEES). CEES developed two mixture proportions with adequate self consolidating properties. One was a stabilizing grout mixture and the other was a bulk fill mixture. These grout mixtures were designed for the closure of 75-ft diameter SSTs. With such a massive pour, one must take into consideration temperature elevations. Research was done to find an efficient method or tool that could be used to predict grout curing temperatures. As a result, ConcreteWorks was selected. This software uses empirical heat of hydration data along with classical heat transfer equations which are explained in detailed in the Appendix A?.

Following waste retrieval the tanks will need to be closed. The current planning basis is to close the tanks by filling them with cementitious material, such as grout.



**Figure 1 - Tank Closure**

## 2. EXECUTIVE SUMMARY

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This research has been supported by the DOE-FIU Science and Technology Workforce Development Initiative Program. During the summer of 2009, a Florida International University (FIU) DOE Fellow (Raul Dominquez) spent 9 weeks doing a summer internship at Columbia Energy and Environmental Services, Inc. under the supervision and guidance of Mr. Colin Henderson, M.S., P.E., Project Manager. The intern's project was initiated on June 22, 2009 and continued through August 21, 2009 with the objective of working on the thermal analysis of the single shell tank closure process while account the variation of materials that compose the two layers of grout that will be used. Objectives include gaining an understanding of the processes involved and calculating the heat of hydration of the grout, understanding and preventing the issues occurring with elevated temperatures and identifying and demonstrating a method to predict temperatures during the grout curing process that takes into account the grout formulations and the physical environment of the buried tank. Software called ConcreteWorks V2 was selected and manipulated to predict temperatures at different grout lift heights. Results indicated that maximum temperatures will not exceed 125°F using Columbia Energy's grout recipe.



### 3. RESEARCH DESCRIPTION

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When filling a mass volume with any cementitious material, one must take into account factors affecting overall quality of the mass pour. American Concrete Institute defines mass concrete as “Any Volume of concrete with dimensions large enough to require that measures be taken to cope with generation of heat from hydration of the cement and attendant volume change, to minimize cracking.” (ACI 116R-00). Rises in temperature can be detrimental to mass concrete structures. One of the greatest concerns in mass concrete placement is the maximum temperature that occurs at the center of the mass. Most specifications usually limit the maximum temperature to 160°F (Gajda, 2006). Elevated temperatures during curing can cause an increase in porosity and a decrease in compressive strength. The other major concern is to exceed maximum temperature deviations. According to standards maximum temperature deviations should not exceed 36°F (Gajda, 2006). High temperature deviations can cause cracking and induce deformation of the structure. The issue with these standards is that they are based on structural concrete. Instead we will be using grout for its self-consolidating properties and there is no temperature limits established for the use of grout in a tank closure application. This research is being conducted to predict curing temperature.

#### 3.1. Selection of Method:

A diligent search was conducted for an efficient method or tool that can be used to predict grout curing temperatures. After analyzing literature, codebooks and multiple software packages ConcreteWorks V2 proved to be the best accessible tool. ConcreteWorks V2 was developed by The Concrete Durability Center of the University of Texas and funded by the Texas Department of Transportation. Texas DOT wanted a simple, user-friendly method or tool for concrete mixture proportioning and thermal analysis (Folliard, 2007). ConcreteWorks provides all that with a variety of design modules for different mass concrete pours. Its versatile mixture proportions and easy to use graphical user interface allowed us to input our specific recipes. ConcreteWorks graphical user interface in the mixture proportion window is demonstrated in Figure 2.

**Mixture Proportion Inputs**

**Mix Proportion Inputs**

Cement Content: 200 lb/yd<sup>3</sup>

Water Content: 371 lb/yd<sup>3</sup>

Coarse Aggregate Content: 00001 lb/yd<sup>3</sup>

Fine Aggregate Content: 2562 lb/yd<sup>3</sup>

Air Content: 17 %

**Supplementary Cementing Materials**

Click on the check to indicate if an admixture is in the mix -

☐ Class C Fly Ash

☒ Class F Fly Ash 390 lb/yd<sup>3</sup> 5 % CaO

☐ Grade 120 Slag

☐ Silica Fume

☐ Ultra Fine Fly Ash

**Chemical Admixture Inputs**

☐ Low Range Water Reducer (Type A)

☐ Mid-Range Water Reducer

☐ Napthalene High-Range Water Reducer (Type F)

☒ Polycarboxylate High-Range Water Reducer (Type F)

☐ Retarder (Type B)

☒ Accelerator (Type C)

Need Help with Chemical Admixture Inputs?

**Mix Proportions (% by weight)**

Pie chart showing the distribution of materials by weight:

- cement: 11.07%
- water: 10.53%
- coarse agg: 5.68%
- fine agg: 72.72%
- c ash: 0%
- f ash: 0%
- slag: 0%
- silica fume: 0%
- ultra fine: 0%

**Calculated Mixture Proportion**

Sacks of Cement/yd<sup>3</sup>: 6.3

Gallons of water/sack of Cement: 7.1

Water/Cement: 1.86

Water/Cementitious: 0.63

Go to Design of Mixture Proportion

Back Next

**Figure 2 - Mixture Proportion GUI**

ConcreteWorks also allows us to input weather data and duration of analysis. Weather data can be selected from a library of meteorological records according to the state and city you run your model in, or you can manually input your own meteorological data. The mechanical properties in Concrete works in a similar fashion, you have the option of manually imputing data or using default values in the ConcreteWorks database. Calculations are then performed using empirical formulas with variables dependent of the mixture proportions that you set. The combination of empirical heat of hydration data and classical heat transfer equation provide reliable temperature predictions.

Validation of the software can be found in a handful of papers. One of which is called “Prediction Model for Concrete Behavior – Final Report”. Researchers compared 12 theoretical models to experimental replicas with temperature differences ranging between 0-6°F.

### 3.2 Modeling of SSTs using ConcreteWorks

Using ConcreteWorks is quick and simple when the inputs are known. A flow chart describing the process of using ConcreteWorks is shown in Figure 3.

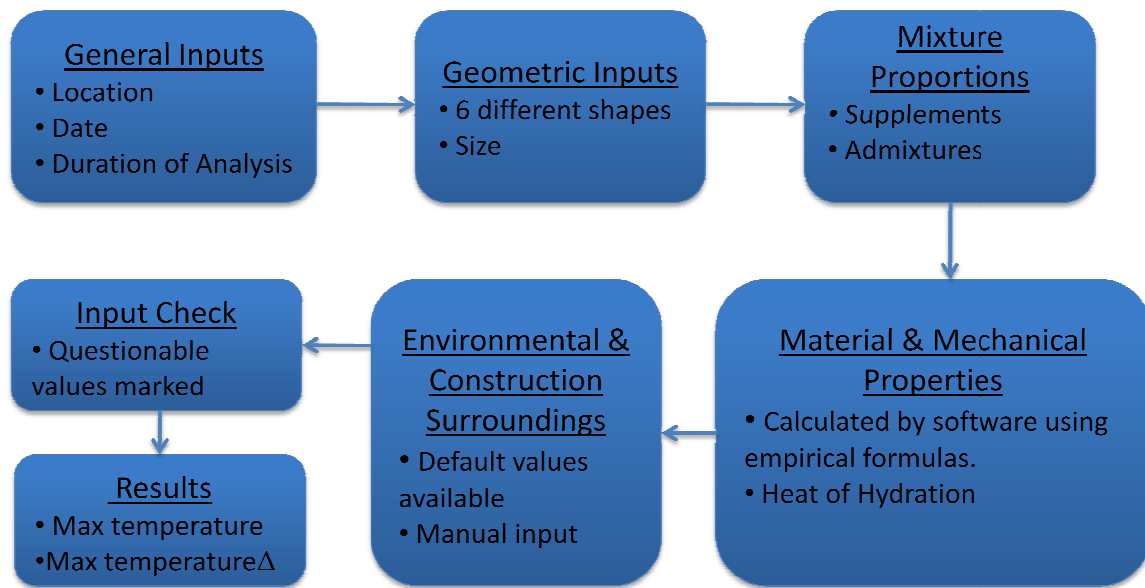


Figure 3 – ConcreteWorks Process Flowchart

Mass concrete in ConcreteWorks can be modeled in six different shapes. All shapes in ConcreteWorks have a number of predefined geometries that can be used to model a mass pour. Different modes of heat transfer are accounted for depending on the geometry selected. From the available geometries, the two shapes that best described the SSTs were selected. The first set of modules were tested in the circular column mode. The advantage of this geometry is that the circular form resembled that of the circular tank. This mode produces 2D values for a horizontal cross-section. Figure 4 shows the heat transfer pathways that are accounted for in the circular column geometry.

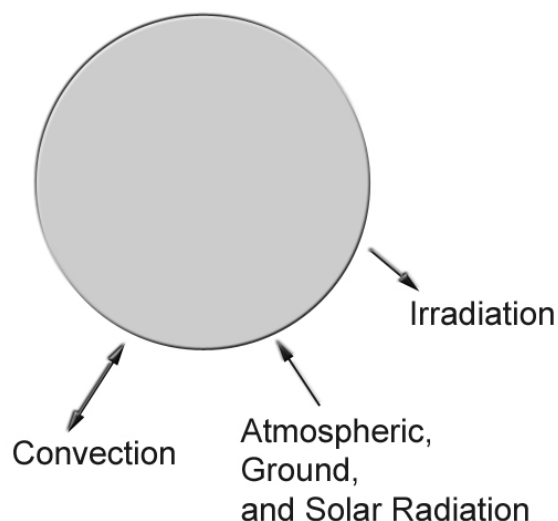
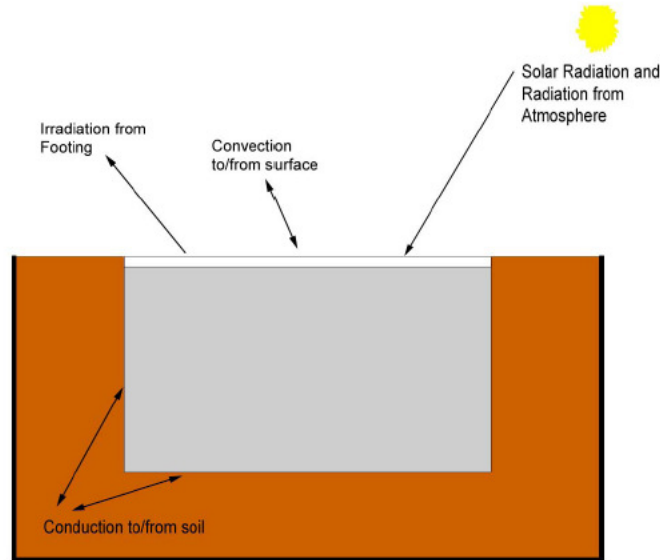


Figure 4 – Circular Column

Figure 5 shows the heat transfer pathways accounted for in the rectangular footing geometry. Rectangular footings were selected because their buried environment and the heat transfer pathways most closely represented the single-shell tank application. Additionally, the rectangular footing provides the ability to produce 3D values for multiple cross-sections in the footing. The disadvantage of this geometry is that the rectangular form may produce greater values for temperature deviations than in a circular tank where the heat is evenly distributed along the circumference of a tank.



**Figure 5 – Rectangular Footing**

The rectangular footing method also allows you to model a curing blanket over your concrete. The footings are modeled at grade level such as our three experimental tanks but this may affect the results because the SST tanks are buried 10 ft below the ground.

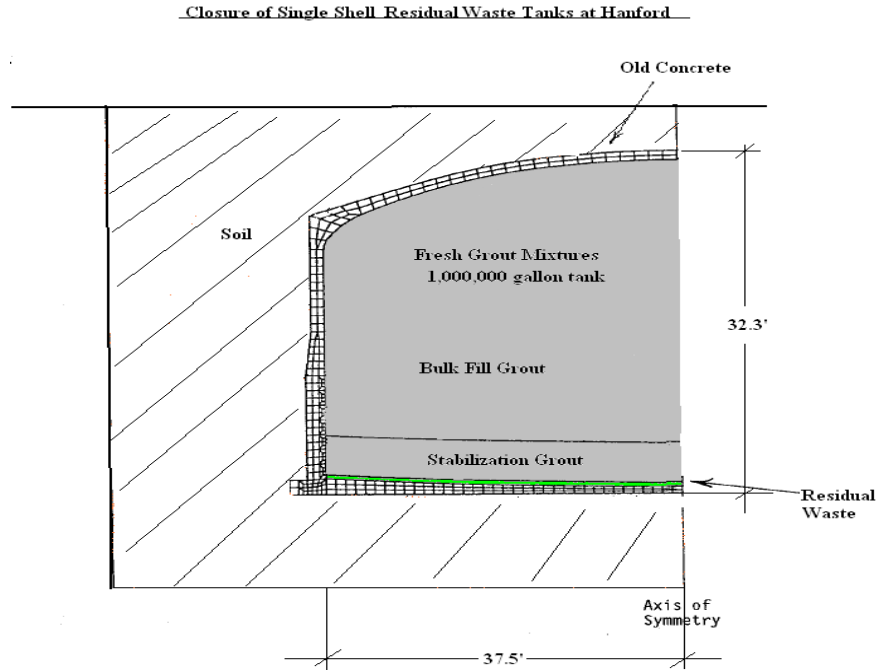


Figure 6 shows a cross section of one-half of a SST in final closure conditions. The first four feet will consist of the stabilization grout used to stabilize any small amounts of residual waste that might remain within tank. Bulk fill grout will fill the majority of the tank.

## 4. RESULTS AND ANALYSIS

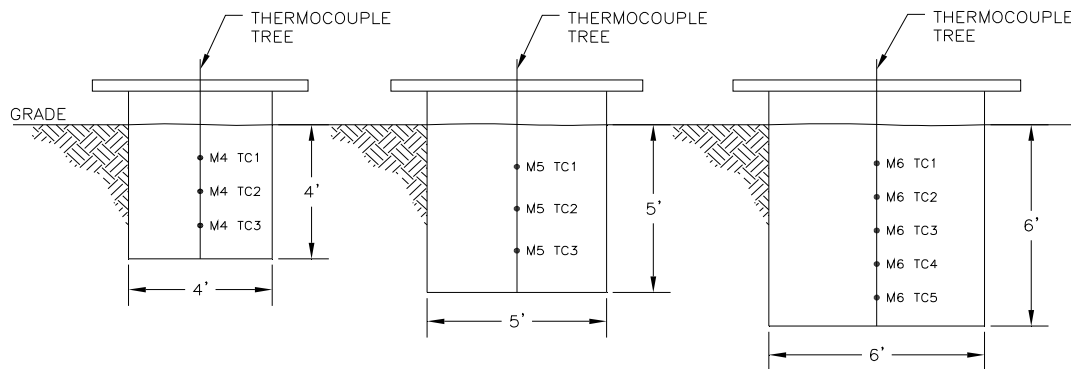
Two grout formulations are planned for use during SST closure. The majority of the tank will be filled with a self-leveling bulk fill mix. Due to possible small amounts of residual waste remaining at time of mass pour, the first mass pour will have a 4ft lift height and will contain granulated blast-furnace slag for its stabilizing properties.

Grout recipes were evaluated and tested by Columbia Energy & Environmental Services, Inc. in RPP-RPT-41550. These recipes were formulated to maximize the use of materials that are locally available. Proportions were adjusted to maximize flow ability, and to minimize shrinkage, bleed water and long term subsidence.

**Table 1 Grout Recipes (RPP-RPT-41550, Grout Optimization Test Report)**

<b>Ingredient</b>	<b>Stabilization Grout 1 CY</b>	<b>Bulk Fill Grout 1 CY</b>
Portland Cement Type I/II (lb) ( $\pm 1\%$ )	118	200
Slag (lb) ( $\pm 1\%$ )	351	0
Pozzolan Class F (lb) ( $\pm 1\%$ )	230	390
Water (lb) ( $\pm 1.5\%$ )	430	371
Sand (lb) ( $\pm 2\%$ )	2,712	2,562
Glenium® 3030 (fl. oz) ( $\pm 3\%$ )	38.5	6
Rheocell 30 (fl. oz) ( $\pm 3\%$ )	0	4

Schematics for Columbia Energy's grout lift height experiment are shown below in Figure 7 (Henderson, 2009). Columbia Energy conducted an experiment with bulk fill grout where they buried three plastic cylindrical tanks at grade level. These tanks were of 4ft, 5ft and 6ft diameters. Their heights were equal to their diameters. They placed thermal couples in the center to record the maximum temperature in the center at different cross-sections.



**Figure 7 - Experimental Data Schematic**

This was done in an attempt to collect temperature data that could be extrapolated to large diameter tanks in order to develop a maximum grout lift height. The results from this test indicate that with increasing size the maximum temperature increases as well. This was not enough to predict temperatures within a 75ft diameter tank. Data from thermocouples 2 and 4 were selected from the 6ft cylinder, to be used for comparison with theoretical values. To test the ability of ConcreteWorks to predict temperatures during grout curing for the selected grout formulations, thermocouple data was compared to modeled data for 2ft and 4ft cross sections of a 6x6x6 ft footing. Model inputs were selected to match the surrounding environment experience in Richland, Washington in late April 2009. Model results demonstrated a close fit to experimental data. Figure 8 demonstrates that maximum theoretical temperature results under predicted maximum experimental temperatures results by a maximum of 10°F. These temperature models were analyzed for a seven day period. The deeper 4ft cross-section demonstrated a better match than the 2ft cross section. This implies that larger modules will have extremely accurate values in the center.

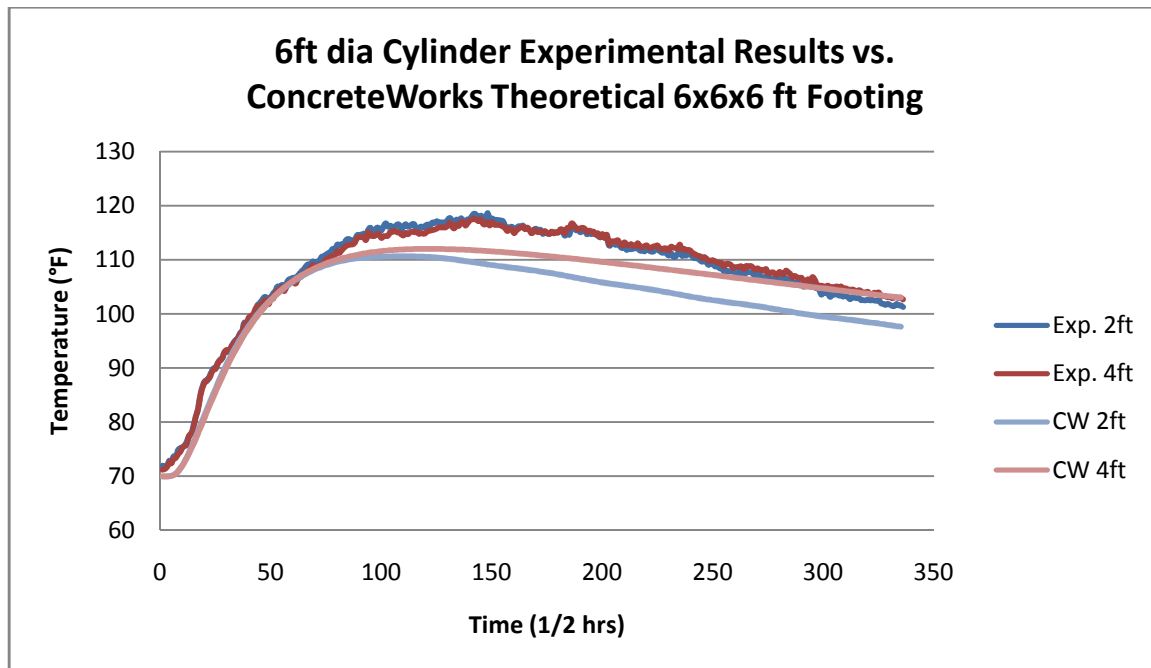


Figure 8 - Validation Graph

Deviations between theoretical and experimental maximum temperatures are shown in Figure 9, demonstrating maximum temperature deviation at the 2ft cross-section. The 4ft cross-section demonstrated a maximum temperature deviation of approximately  $6^{\circ}\text{F}$ . This amount of error is comparable to amounts found in literature (Folliard, 2007).

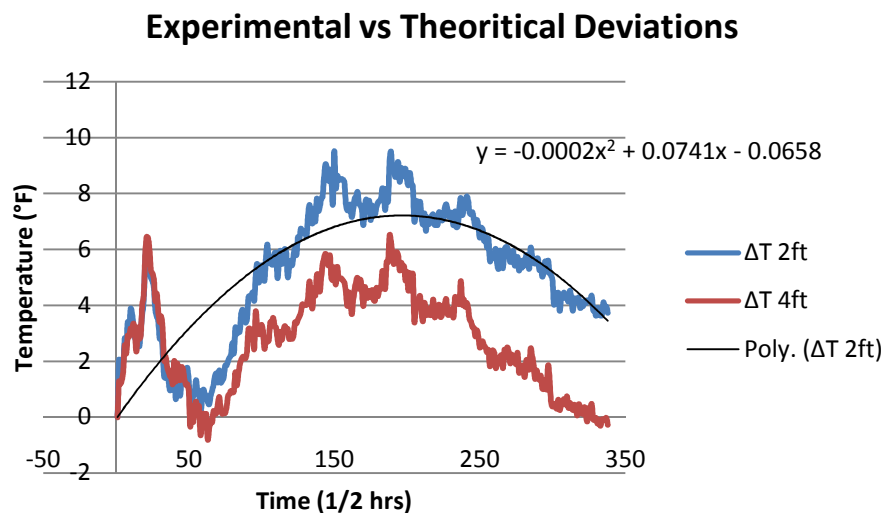


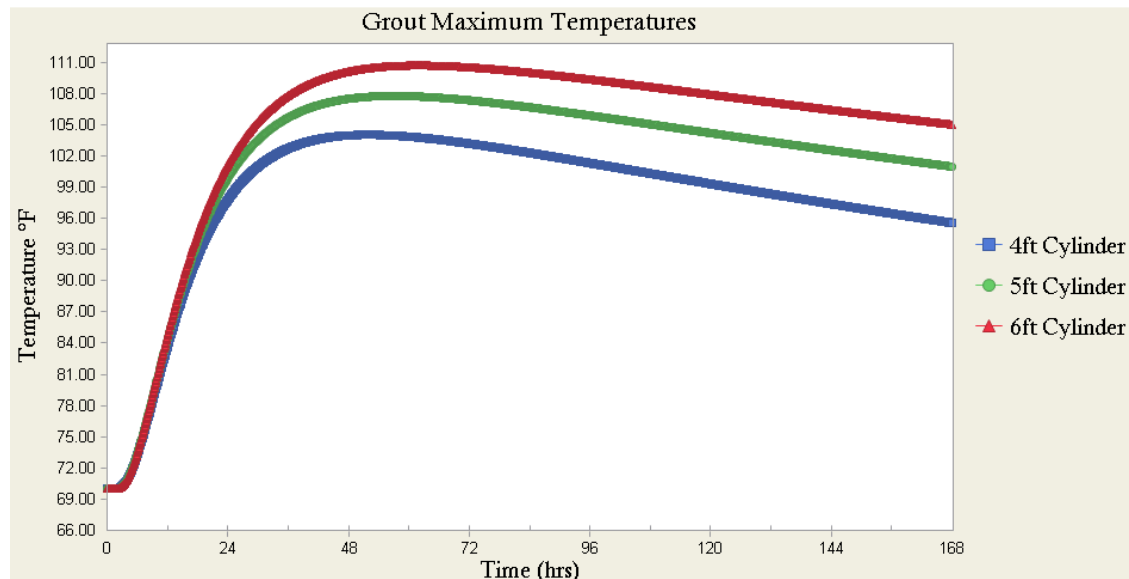
Figure 9 - Validation Deviations

## 4.1 Circular Columns

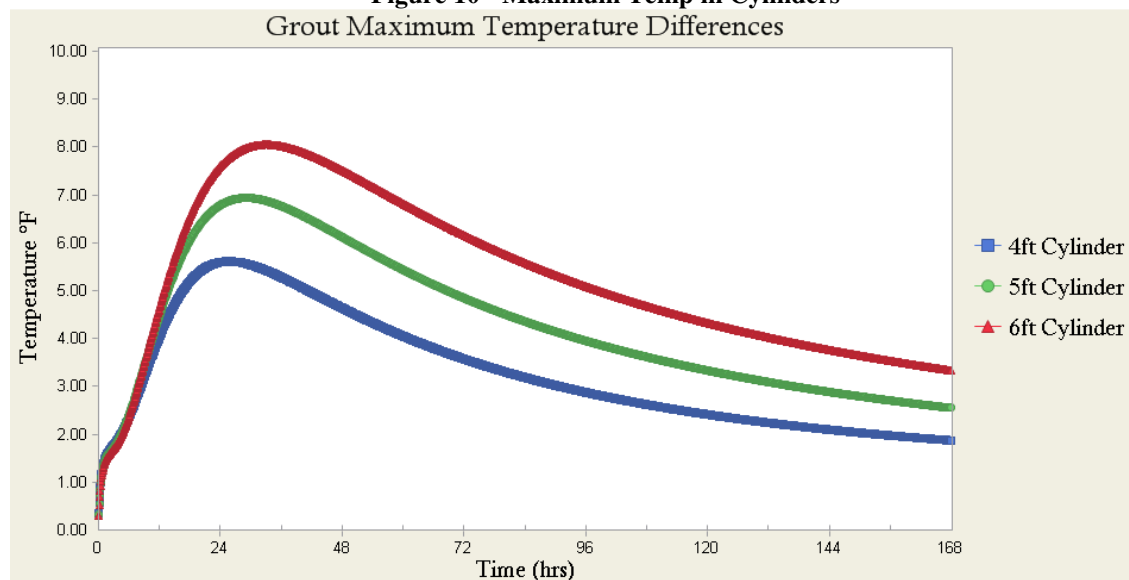
Three circular column models were generated at 4ft, 5ft and 6ft diameter. Results obtained behave similar to experimental results. Showing an increase in temperature according to increase in diameter. Maximum temperature differences between the hottest point and the



coldest point in the cylinder stayed relatively low but still increased with diameter as shown in Figures 11 and 12.



**Figure 10 - Maximum Temp in Cylinders**



**Figure 11- Maximum Temperature Deviations in Cylinder**

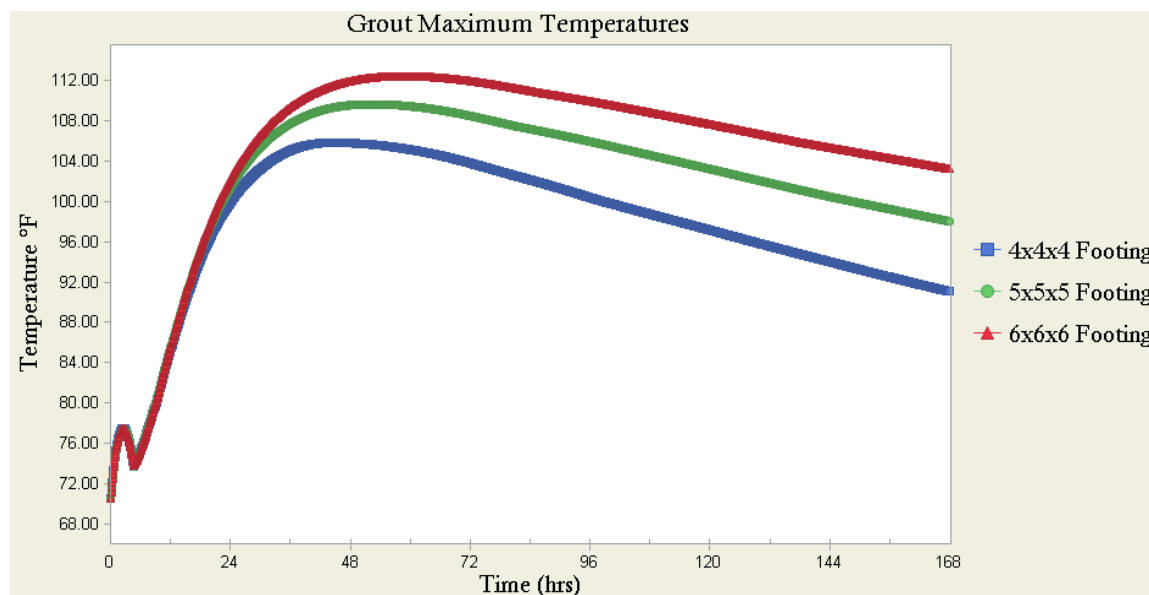
When maximum temperatures are compared to experimental data an average under prediction of about 6°F is observed as shown in Table 2. The circular column mode of ConcreteWorks does not take into account for heat transfer perpendicular to the horizontal cross section. While this model is accurate for small diameters it will lose accuracy for larger diameters where vertical heat transfer plays a larger role.

**Table 2 - Circular columns**

<b>Cylinder (diameter)</b>	<b>CWs Max Temp (°F)</b>	<b>Field Max Temp (°F)</b>	<b>Deviation (°F)</b>
4 ft	104	106	-2
5 ft	107	111	-4
6 ft	110	121	-11

## 4.2 Rectangular Footings

Three sizes of rectangular footings were modeled simulate the problem. Figure 12 demonstrates their maximum temperatures. Very similar behavior was observed between the circular columns models and rectangular footing models for maximum temperature. That was not the case with maximum temperature differences as shown in Figure 14. Significant fluctuations occurred in maximum temperature differences that might be a result of the fact that the footing is at grade level where its minimum temperature is highly affected by ambient conditions. There is also large over prediction in temperature differences. This is a result of the rectangular geometry where the corners are exposed and cool quickly. In a real tank environment none of these two issues will be encountered.

**Figure 12 - Maximum Temp in Rectangular Footings**

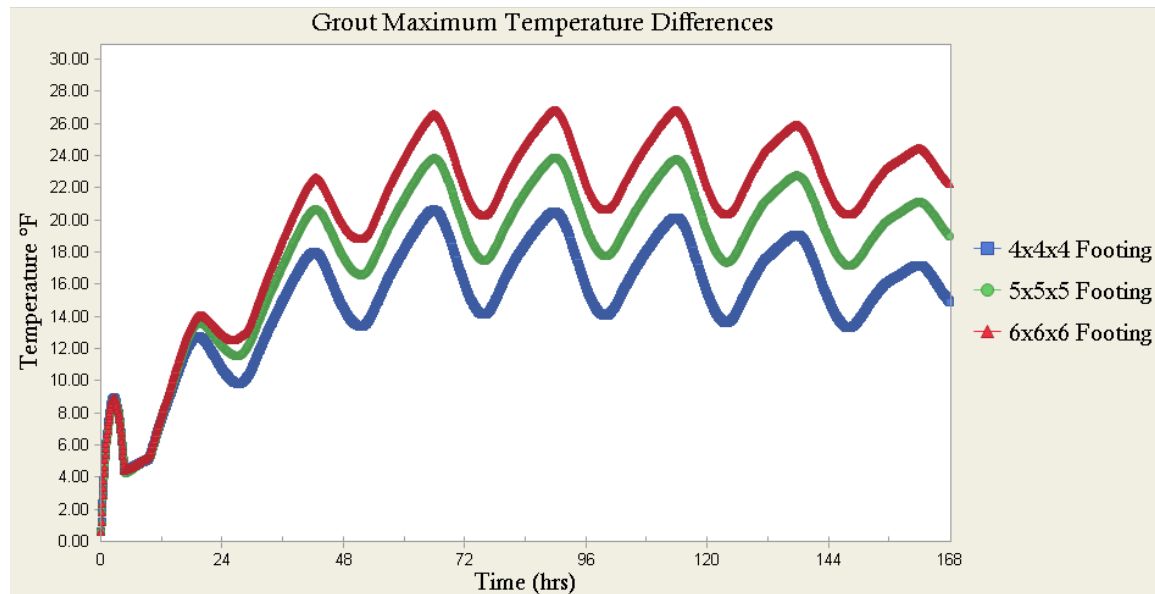


Figure 13 - Maximum Temperature Deviations in Footings

Table 3 - Rectangular Footings

Footing (dimensions)	CWs Max Temp (°F)	Field Max Temp (°F)	Deviation (°F)
4x4x4 ft	105	106	-1
5x5x5 ft	109	111	-2
6x6x6 ft	112	121	-9

Maximum temperatures for rectangular footings are compared to maximum temperatures collected from the grout lift height experiment in table 3. With average under predictions of 4°F, rectangular footings proved to be the better shape for our purposes.

### 4.3 Large Footings

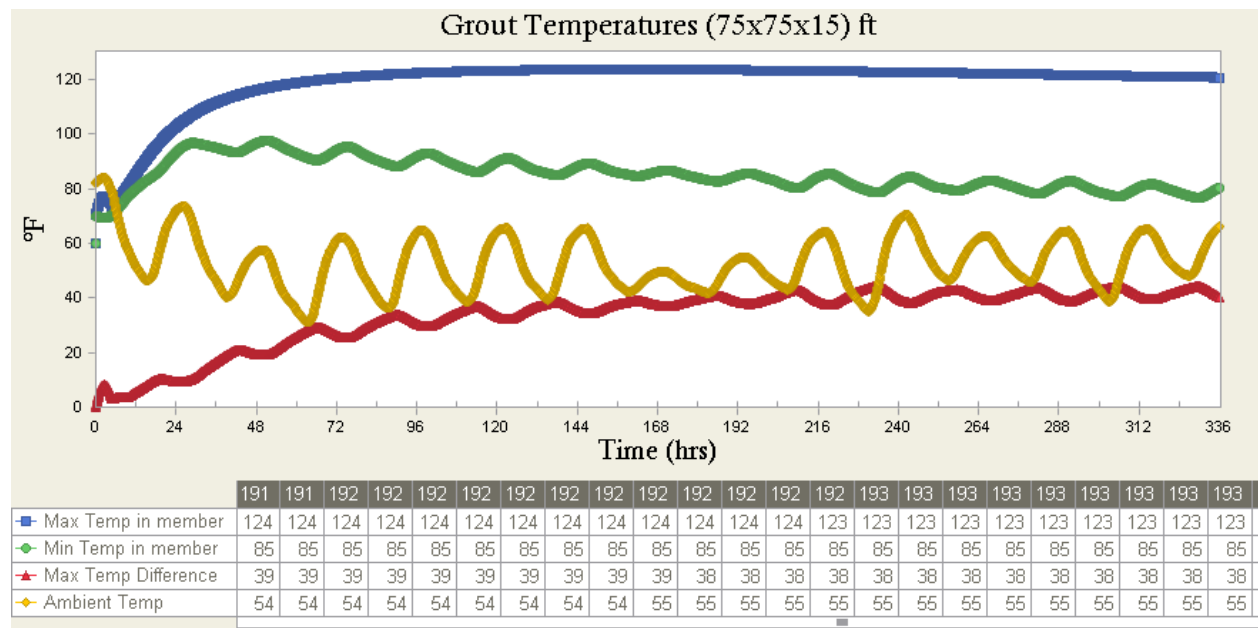
After deciding that the rectangular footing geometry was the best representation of the tank closure problem, large scale models were run to simulate the actual size of a SST. One issue we encountered is that ConcreteWorks allows you to run the thermal analysis for only 14 days. Two weeks was not enough time for us to see the maximum temperature in the 20 ft and 50 ft deep simulated geometries. The results graphs for both of these geometries indicate that the temperature is asymptotically approaching a maximum. The 15 ft deep model did reach a peak temperature and begin to decrease within the 14 day window.

[illegible]

**Grout Temperatures (75x75x20)ft**

	333	333	333	333	333	333	333	333	333	333	333	333	334	334	334	334	334	334	334	334	334	334	334	334
Max Temp in member	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125
Min Temp in member	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78	78
Max Temp Difference	47	47	47	47	47	47	47	47	47	47	47	47	47	47	47	46	46	46	46	46	46	46	46	46
Ambient Temp	61	61	61	61	62	62	62	62	62	62	62	62	63	63	63	63	63	63	64	64	64	64	64	64

**Figure 15 - 75x75x20 ft Footing**

**Figure 16 - 75x75x15 ft Footing**

The 75x75x50 ft footing does not reach a maximum temperature within the 14 days, but it did exceed maximum temperature difference. The 20 ft lift height does not reach maximum temperature but it does maintain a steady temp of 125°F for over 7 days. The 15 ft lift height demonstrated a maximum temperature and exceeded maximum temperature difference. Results for all three lift heights can be observed within table 4.

**Table 4 - Lift Heights**

Lift Height(ft)	Max Temp Recorded (°F)	Max Temp Difference(°F)
15	123	44
20	125	47
50	126	49

The temperature data was exported from ConcreteWorks in an excel format and inputted into Sigma plot to obtain 3D contour graphs the maximum temperature with respect to time and depth of cross section.

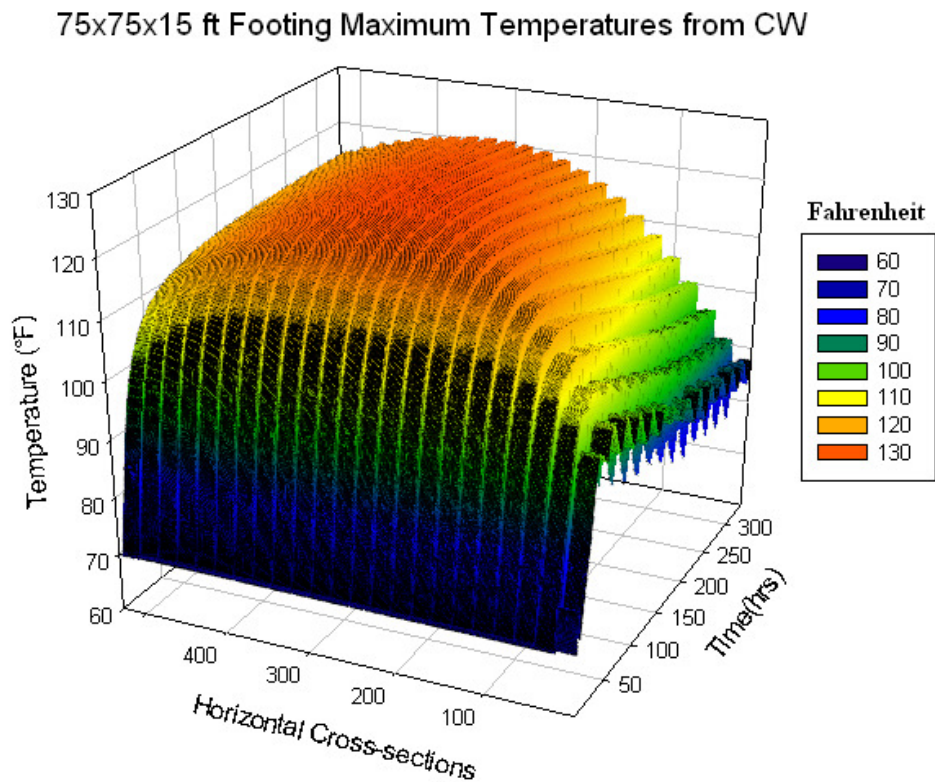


Figure 17 - 3D Graph of 15ft Lift Height

Three dimensional plots demonstrate plateauing behavior proving that maximum temperatures will not increase by much after the 14 day analysis.

## 75x75x50 ft Footing Maximum Temperatures from CW

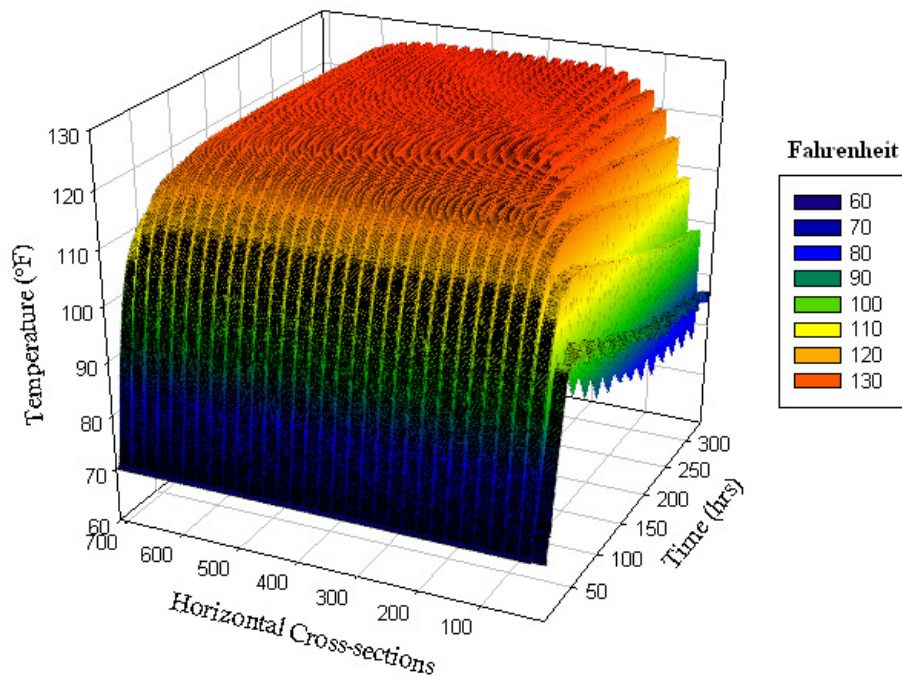


Figure 18 - 3D Graph of 50 ft Lift Height

Large Footings were compared with two excessively large footings. The two extra large footings overlapped the 75x75x50. This may mean that once you exceed a certain size there is really no change in temperature.

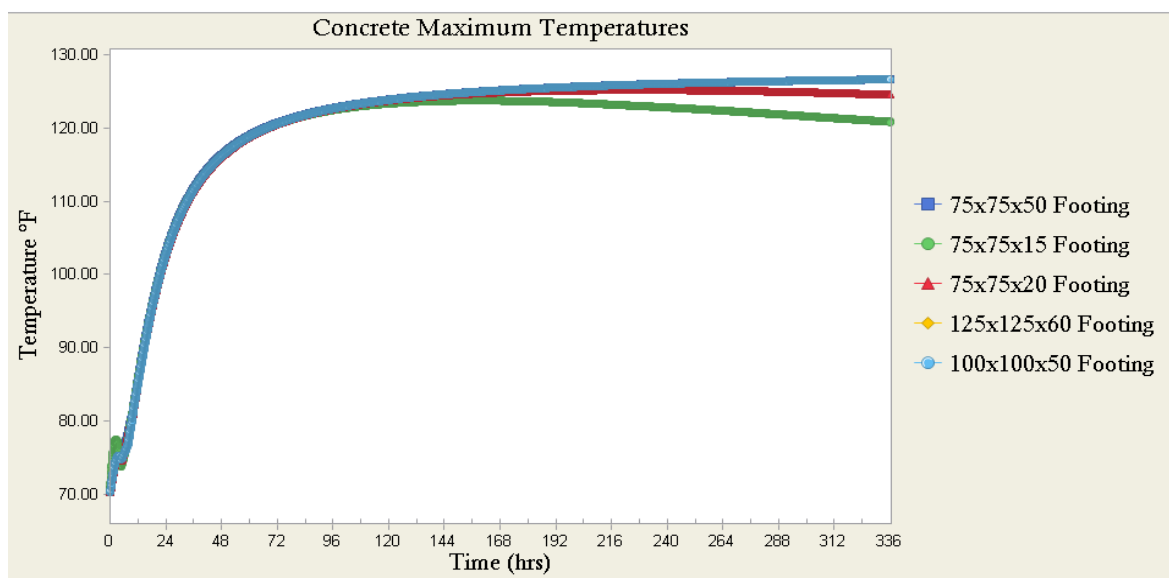


Figure 19 - Additional Models for Larger Volumes

## 5. CONCLUSION

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Mass grout filling is a viable option for single shell tank closure. ConcreteWorks is a reasonable tool for predicting temperatures for grout pours, based on evaluation the rectangular footing best represented the tank closure situation, and the max temperatures predicted were less than 130 °F. Due to the small amount of cement contained in the grout formulation designed for use during bulk filling of the tanks, elevated temperatures will typically not exceed levels of concern. The Circular column mode and rectangular footing mode were both tested for compatibility to the single shell underground tank environment. Rectangular footings provided best results for maximum temperature. Maximum temperature values plateauing prove that grout lift height will not be a concern or limitation. Theoretically it would be possible to pour a constant flow of grout into the tanks without having to worry about maximum temperature limitations.



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## 7. APPENDIX A. Heat Transfer Theory

This research was conducted using carefully selected software called ConcreteWorks V2. This software uses empirical heat of hydration data along with classical heat transfer equations. ConcreteWorks will be explained in greater detail in the Selection of Method section.

Classical heat transfer equations are fundamental in predicting temperature behavior, beginning with the basic Heat Flux equation. Heat flux is the measurement of the heat that propagates through a particular median. To demonstrate this heat transfer principle we will analyze a control volume of a simple cube.

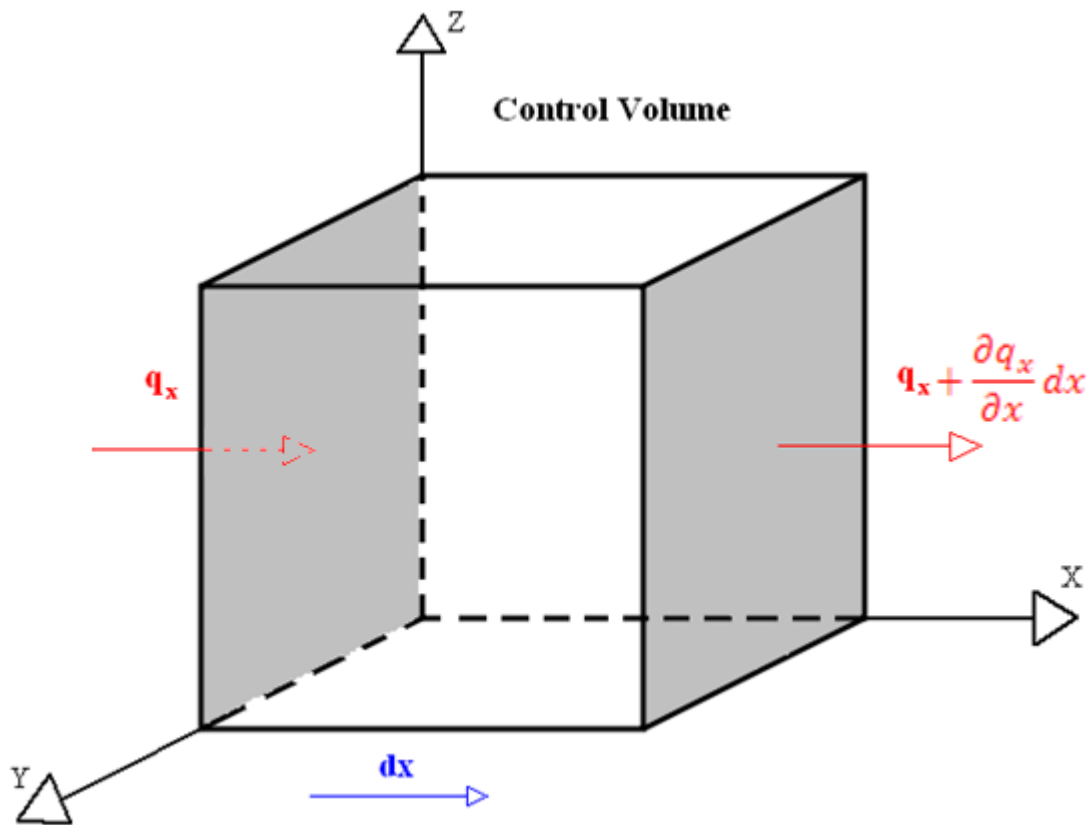


Figure 200 – Control Volume

*Heat Flux in the x – direction is defined as*

$$q_x = -kA \frac{\partial T}{\partial x} \quad (1)$$

$k$  = Thermal Conductivity

$A$  = Cross Sectional Area/Plane Perpendicular to Directional Vector

$\frac{\partial T}{\partial x}$  = Change of Temperature with respect to the  $x$  axis

At first we analyze the heat transfer occurring in the  $x$  direction. Heat flux is always perpendicular to the cross-sectional plane. The change in heat flux through the control volume is equal to the partial differential of  $q_x$  with respect to  $x$  times  $dx$ . Taking into account the law of Conservation of Energy we obtain an energy balance equation.

*Energy Balance*

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st} \quad (2)$$

$E_{in}$  = Thermal Energy entering control volume

$E_{out}$  = Thermal Energy leaving control volume

$E_g$  = Thermal Energy generated withing control volume (Heat of Hydration)

$E_{st}$  = Change in Thermal Energy Stored in Volume

Plugging in the Heat Flux equation into our energy balance and dividing out the dimensions of the control volume provides us with the heat diffusion equation.

*Heat diffusion equation derivitation*

$$-q_x + \left( q_x + \frac{\partial q_x}{\partial x} dx \right) + q dx = \rho C_p \frac{\partial T}{\partial t} dx$$

↓

$$\left( \frac{\partial \left( -kA \frac{\partial T}{\partial x} \right)}{\partial x} dx \right) + q dx = \rho C_p \frac{\partial T}{\partial t} dx$$

↓

$$\left( \frac{\partial \left( -k \frac{\partial T}{\partial x} \right)}{\partial x} dx dy dz \right) + q dx dy dz = \rho C_p \frac{\partial T}{\partial t} dx dy dz$$

↓

$$\frac{\partial}{\partial x} \left( -k \frac{\partial T}{\partial x} \right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t} \quad (3)$$

By expanding equation (3) to take into account for heat transfer in all three dimensions you produce the Heat diffusion equation which is used by ConcreteWorks.

*Heat diffusion Equation (Incropera, 1996)*

$$\frac{\partial}{\partial x} \left( -k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( -k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( -k \frac{\partial T}{\partial z} \right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t} \quad (4)$$

$\dot{q}$  = rate of heat generation

$\rho$  = Density

$C_p$  = Specific heat capacity

$\frac{\partial T}{\partial t}$  = Temperature with respect to time

These parameters are obtained from empirical formulas used within the Concreteworks. Heat of hydration is the heat developed by the exothermic chemical reaction between cement, slag, fly-ash and water occurring during the curing process of concrete. Heat from hydration is mainly dependent upon the chemical composition of the mixture and is usually not a concern unless undergoing a mass pour.